APPLICATIONS OF THE ELECTROMECHANICAL ANALOGIES AND THE EQUIVALENT CIRCUIT IN ULTRASONIC PIEZOCERAMIC MICROACTUATION

DR. ENG. MIRCEA IGNAT - INCDIE

Key words: Microelectromechanics, Microactuators, Analogies, Equivalent circuit.

The paper presents theoretical and experimental aspects about the electro-mechanical analogies and the equivalent circuit in ultrasonic piezoceramic actuation (Mason’s equivalent circuit).

Essentially, the electromechanical analogy represents comparison between the electric circuit; voltage – force, current – speed, inductance – mass, capacitance – compliance. The analogy enables the relations in an electrical circuit to be formed in familiar terms of mechanical phenomena. Having developed electric-circuit theory to a high degree, it becomes desirable to use a similar analogy to simplify the treatment of mechanical systems.

It presents the case of ultrasonic piezoceramic micromotors and microactuators where the input parameters are electric parameters and the output of microactuator system are mechanical parameters. Is analyzed the equivalent circuit of a microactuator and the fundamental equations:
- fundamental equations of inverse effect;
- specific circuit equation;
- equation of motion,
and the relations between the electric and micromechanical parameters function by the longitudinal effect, multilayer piezoelectric ceramic, micromechanical resonance.

1. INTRODUCTION

The electromechanical analogies represent an analytically analysis method of the mechanical systems [1,2,4,6,7,8].

There are some interesting analogies between mechanical phenomena in elastic bodies and the electrical effects in dielectrics. Since piezoelectricity deals with a system to what extent the relations can be made symmetrical... also in how correspondences thus disclosed may be regarded as of more than merely formal significance.

It can be foreseen at the outset that full symmetry in the mechanical formulation is impossible, for the following reasons [4,5,6]:
- Elastic stresses and strains are tensors while electric fields and polarization are vectors, the field being characterised by a potential gradient.
- Across the two media it is normal elastic stress that is continuous while in the electric case the continuity is in the electric displacement and not in the field. Furthermore an elastic stress is defined as force per unit area while the electric stress or field strength is force per unit charge.

The electromechanical analogies theory includes two kinds and six types [2]:

<table>
<thead>
<tr>
<th>Kind I analogies</th>
<th>Kind II analogies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical impedance type</td>
<td>Mechanical admittance type</td>
</tr>
<tr>
<td>Specific impedance type</td>
<td>Specific admittance type</td>
</tr>
<tr>
<td>Acoustic impedance type</td>
<td>Acoustic admittance type</td>
</tr>
</tbody>
</table>
An interesting field of electromechanical analogies applications is the field of microelectromechanical ultrasonic piezoceramic actuators (Actuators represents “a mechanism to activate process” and actuate “to put into motion or mechanical action”[10]).

In microactuator electromechanics very important become the kind I analogies, mechanical impedance type where:

<table>
<thead>
<tr>
<th>Mechanical parameters</th>
<th>Electric parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force F</td>
<td>Electric voltage U</td>
</tr>
<tr>
<td>Speed v</td>
<td>Electric current I</td>
</tr>
<tr>
<td>Mechanical impedance;</td>
<td>Electric impedance ;</td>
</tr>
<tr>
<td>$Z_m = \frac{F}{v}$</td>
<td>$Z_e = \frac{U}{I}$</td>
</tr>
<tr>
<td>Mass m [kg]</td>
<td>Inductance L</td>
</tr>
<tr>
<td>Compliance $C_m$</td>
<td>Capacitance $C_e$</td>
</tr>
<tr>
<td>Mechanical resistance $R_m$</td>
<td>Electric resistance R</td>
</tr>
</tbody>
</table>

An ultrasonic microactuator is an microelectromechanical system which uses ultrasonic vibration – a type of elastic vibration – to obtain a driving force, which trend drives an element (mark) using friction. Operating principle of the ultrasonic piezoelectric actuation or piezoelectric micromotors [9] is presented in fig.1.

The main methods of producing displacement motion (microelectromechanical actuation) in piezoceramic converter are [9]:
- Use of a flexural progressive wave.

It is well known that a Rayleigh wave propagation along the surface of a semi-infinite medium (medium I) forms the counterclockwise elliptical motion of a displacement $u_0$ and $\omega_0$, shown in fig.2. These elliptical displacement motion are typical in a linearly vibrating medium.
Excitation by rotation of resonance modes.
Degenerate vibration mode exist in the disk or ring vibrators. The disk vibrates in the direction perpendicular to its surface. The resonance modes are represented as $B_{mn}$ modes ($m$ - is the nodal circles and $n$ - that of nodal diameters (to see fig.3).

$$B_{mn} (m = 1, n = 3)$$

Fig. 2 Elliptical motion by surface wave to piezoceramic medium.

- Ultrasonic microactuators and micromotors are friction-driven and their rotational speed decreases with increasing load. This load characteristics is similar to that of the electromagnetic micromotor or actuator. The ultrasonic micromotors and microactuators advantages include:
  - both high torque and high efficiency at low speed;
  - the possibility of direct drive;
  - ability to maintain its own position;
  - good control characteristics at start and stop;
  - simple structure and flexibility of shape;
- no restriction through induction (e.g. electromagnetic);
- silent operation.

Its disadvantages include:
- need for electrical source of high frequency;
- inevitable wear and tear of the contact surfaces;
- expensive electrical source and piezoelectric elements.

In fig.4 is represented the structure of an ultrasonic microactuator (micromotor).

Fig.4. Specific ultrasonic micromotor structure: 1. rotor, 2. element of vibration transmission, 3. electrode, 4. stator piezoceramic converter, 5. electrode, 5. mechanical support of piezoceramic converter, 7. resor, 8 control element of stator–rotor pressure, 9 nut, 10 electric isolator.

2. PIEZOELECTRIC FUNDAMENTAL EQUATIONS

In the electromechanical engineering treatment of a piezoelectric conversion and actuation, the following equations, including the linear piezoelectric constitutive equations, must be taken into considerations [9,11]:

**Equation of motion**

\[ T_{ij,j} = \rho \ddot{u}_j \quad (i, j = 1 - 3) \]  

\[ (1) \]

**Electrical conditions**

\[ D_{ij} = 0 \quad (\text{div}D = 0) \]

\[ (2) \]

\[ E_k = -\varphi_k \text{ (relations between electric field and potential)} \]

\[ (3) \]
Linear piezoelectric constitutive equations:

\[ T_p = C_{pq}^E S_q - e_{kp} E_k \]
\[ D_i = e_{ik} S_k + \varepsilon_{ik}^E E_k \]  \hspace{1cm} (p, q = 1 - 6, k = 1 - 3)  \hspace{1cm} (4)

Strain and displacement:

\[ S_q = u_{k,k} \quad \text{(longitudinal strain: } S_1, S_2, S_3 \text{)} \]  \hspace{1cm} (5)
\[ S_q = u_{k,l} + u_{l,k} \quad \text{(shear strain: } S_4, S_5, S_6 \text{)} \]  \hspace{1cm} (6)

Here \( T_{i,j} \) is the stress tensor and corresponds to the practical stress \( T_p \) of engineering notation, while \( D_{i,j} \) is a convenient expression meaning \( D_{i,j} = D_{i,1} + D_{j,2} + D_{3,3} \) and \( i \) signifies differentiation, \( \rho \) density, \( e_{ik} \) is the piezoelectric stress constant, \( \varepsilon_{ik}^E \) is the dielectric constant. In medium I is realised an electromechanical conversion between the electric input parameters (U,I,frequency) and piezoceramic and dielectric material parameter and results an actuation characterised by the output parameters (F,v).

3. EQUIVALENT CIRCUIT OF AN ULTRASONIC MICROACTUATOR

The linear piezoelectric constitutive equations of equations (4) are expressions using mechanical strain and stress, as well as electrical displacement and field; therefore they can be called “microscopic” expressions of piezoelectric vibration. On the other hand, macroscopic expressions are necessary in the electrical equivalent circuit of a piezoelectric convertor, because the circuit is usually expressed by the voltage, current, etc [8,9,11]. Without having into considerations many theoretical aspects of piezoceramic ultrasonic microactuation, in fig.3 is presented the equivalent circuit of an ultrasonic piezoceramic microactuator, where \( A \) is the force factor which relates the electrical arm to the mechanical arm, \( U/I \) are voltage and electric current, \( I_v \) motional current, \( F \) and \( v \) the external force and velocity of a piezoceramic convertor (microelectromechanical actuation), \( C_d, R_d \) are the component of damped admittance and \( Z_m (R_m, C_m, L_m) \) is the mechanical impedance of the piezoceramic convertor.
The equation of equivalent circuit are:

\[ AU = F + R_m v \]  \hspace{1cm} (7) \]

\[ I = Y_d U + Av \]  \hspace{1cm} (8) \]

\[ Y_d = \sqrt{\frac{1}{R_d^2 + C_d^2 \omega^2}} \]  \hspace{1cm} (9) \]

\[ Z_m = \sqrt{R_m^2 + \left( L_m \omega - \frac{1}{C_m \omega} \right)^2} \]  \hspace{1cm} (10) \]

To simplify the equivalent circuit in practical situations, \( R_d \to \infty \). The equivalent stiffness \( K = 1/C_m \) is expressed by the resonance frequency:

\[ f_r = \frac{1}{2\pi} \sqrt{\frac{K}{M}} \]  \hspace{1cm} (11) \]

The equivalent mass \( M = L_m \) is calculated from this equation:
\[ \frac{1}{2} M \left\{ v \left( \frac{1}{2} l \right)^2 \right\} = \int_{-\frac{l}{2}}^{\frac{l}{2}} v(z)^2 S dz \]

\[ v(z) = \frac{du(z)}{dt} \]  \hspace{1cm} (12)

where \( u(z) \) is the deformation of the amplitude of piezoceramic stator, namely the vibration mode (with \( z \) coordinate) and \( l \) the length of piezoceramic convertor. Force factor \( A \) can be obtained from the ratio between the vibration velocity and the input voltage. The boundary condition is no preload (no rotor) \( F = 0 \) and:

\[ A = \frac{R_m v}{U} \]  \hspace{1cm} (13)

Force factor can be obtained deductively using the piezoelectric equation with complication relations for different electromechanical structure and function by the piezoceramic parameter [12]. To mechanical resonance (parameters \( R_m, C_m, L_m \)) in phasorial representation is showed in fig.6. This is a functional state of ultrasonic piezoceramic actuator.

Fig.6. Ultrasonic mechanic phasorial representation to equivalent circuit.

\( F_L, F_C, F_R \) are the microforce phasors and correspond to \( R_m, C_m, L_m \).
4. CONCLUSIONS

The paper attempts to attract attention to an important paradigm for new electromechanical field: electromechanical analogies with a study of case in equivalent circuit of ultrasonic microactuator. There are reported interesting researches in arterial modelling [13] or mechanical fluid, yet with many restraint in engineering education and forming.

References