FEM Analysis of the Third Order Harmonics of a Two-Phase Induction Machine

Alecsandru SIMION, Leonard LIVADARU
“Gh. Asachi” Technical University of Iași, Faculty of Electrical Engineering, Bd. D. Mangeron, Nr. 51-53, 700050 Iași, Romania; asimion@ee.tuiasi.ro

Dorin LUCACHE, Emanuel ROMILA
“Gh. Asachi” Technical University of Iași, Faculty of Electrical Engineering, Bd. D. Mangeron, Nr. 51-53, 700050 Iași, Romania

Abstract. The paper presents a comparative analysis of the space harmonics developed by a two-phase induction machine with common construction (two variants) and a variant with an original special geometry. The results put in view that for the new variant, the third order harmonics that have a negative influence, practically vanish in condition of low saturation operation. The paper presents also the results corresponding to a saturated regime.

1 Introduction
The problem of the high order field harmonics of electrical machines, mainly A.C. machines, has represented a special issue for the specialists since the last century [1, 2, 3...]. The reason consists in their negative effects especially upon the energetical performance. The negative influences of the high order harmonics on the operation of the induction machines can be briefly enumerated as follows: - higher asynchronous and synchronous parasitic torques that involves a smaller output torque and starting difficulties; - increase of vibrations and noise level during operation; - rising of iron losses mainly corresponding to stator and rotor teeth and a decrease of the efficiency as consequence; - increase of the differential dispersion coefficient (from tooth to tooth) and consequently a rising of the total dispersion coefficient of the windings.

The three-phase induction machines have no third order harmonics due to the star-connection of the primary winding. The 5th and the 7th order harmonics can be diminished by using a proper diminution of the winding pitch: \( y = 4/5 \tau \) and \( y = 6/7 \tau \) respectively (\( y \) – winding pitch, \( \tau \) - polar pitch). The high order harmonics can also be decreased by using skewed rotor slots (mainly for cage windings) or/and adopting proper numbers of slots on stator and rotor (for the wound-rotor machines). Since the influence of the harmonics decrease with their frequency, it is taken into account only the problem of diminution of the harmonics under the 11th order.

The two-phase machines have a particular problem with the 3rd order harmonics since the connection of the phases has no influence upon them. This paper deals with the study of the methods that are capable to eliminate the 3rd order harmonics created by the two-phase induction machines with identical stator windings and cage rotor.

2 The magnetic field created by a two-phase stator winding

2.1 The magnetic field created by the fundamental
Fig. 1a presents a machine with a four-pole two-phase stator winding. The two symmetrical current corresponding to the two phases (A-X and B-Y) are:

\[
I_{AX} = I_m \sin \omega t, \quad I_{BY} = I_m \sin(\omega t - \pi / 2) = -I_m \cos \omega t, \quad I_m = I \sqrt{2}
\]  
(1)
Under a certain direction shifted in phase against the horizontal with $\alpha$, the air-gap fundamental wave created by A-X phase is:

$$B_{dAX}(\alpha,t) = B_{1\text{max}} \sin \omega t \cos p \alpha = \frac{1}{2} B_{1\text{max}} \sin(\omega t - p \alpha) + \frac{1}{2} B_{1\text{max}} \sin(\omega t + p \alpha)$$  \hspace{1cm} (2)

where:

$$B_{1\text{max}} = \frac{4}{\pi} B_{20} k_{q1} = \frac{4}{\pi} \mu_0 W I \pi k_{q1} = \frac{4 \sqrt{2}}{\pi} \mu_0 W I \cdot k_{q1} = 1.8 \cdot \mu_0 W I \cdot k_{q1}$$  \hspace{1cm} (3)

In the same direction, the fundamental created by the B-Y phase is:

$$B_{dBY}(\alpha,t) = \frac{1}{2} B_{1\text{max}} \sin(\omega t - p \alpha) + \frac{1}{2} B_{1\text{max}} \sin(\omega t + p \alpha - \pi)$$  \hspace{1cm} (4)

The resultant magnetic field results as a superposition of the effects (when the saturation is neglected):

$$B_{dAX}(\alpha,t) = B_{dAX}(\alpha,t) + B_{dBY}(\alpha,t) = B_{1\text{max}} \sin(\omega t + p \alpha)$$  \hspace{1cm} (5)

It results a forward circular rotating field that has a constant speed of:

$$\Omega = \frac{d\alpha}{dt} = \frac{\omega}{p}$$  \hspace{1cm} (6)

A geometrical signification [7] can be developed (Fig. 1b). The flux density created by the A-X phase has two rotating components: $\overline{B}_{dAX}$ - forward component and $\overline{B}_{dAX}$ - backward component. At a certain moment, they get in phase. Let’s consider it as reference. The flux density created by the B-Y phase (Fig. 1c) has two similar components: $\overline{B}_{dBY}$ and $\overline{B}_{dBY}$. They acts on the same but opposite directions, at the above considered moment, since their sum is to be zero. By superposing the two figures, it results a circular rotating direct field equal to $\overline{B}_{dAX} + \overline{B}_{dBY}$.

2.2 The field created by the high order space harmonics

Each of the two windings creates magnetic fields that can be expressed as Fourier series [3]. When the currents are sinusoidal, the high order space harmonics can be analyzed separately and finally superimposed. Generally, for $v$ - odd, the resultant magnetic field is:

$$B_{derv}(\alpha,t) = B_{v\text{max}} \sin \omega t \cos v \alpha + B_{v\text{max}} \sin(\omega t - \pi/2) \cos(v \alpha - v \pi/2) =$$

$$= \frac{1}{2} B_{v\text{max}}[\sin(\omega t + v \alpha) + \sin(\omega t - v \alpha) + \sin(\omega t + v \alpha - v + 1/2 \pi) + \sin(\omega t - v \alpha + v - 1/2 \pi)]$$  \hspace{1cm} (7)

There are two situations:
\[ B_{\text{gec}}(\alpha, t) = B_{3\text{max}} \sin(\alpha + \nu \omega t) \]  

which means backward rotating magnetic fields with a speed of:

\[ \frac{d\alpha}{dt} = \Omega_{3} = -\frac{\omega}{3\nu} \]  

\[ \nu = 3, 5, 7, 11, ... \]

\[ B_{\text{gec}}(\alpha, t) = B_{3\text{max}} \sin(\alpha - \nu \omega t) \]  

which means forward rotating magnetic fields with a speed of:

\[ \Omega_{3} = \frac{\omega}{3\nu} \]  

\[ \nu = 5, 9, 13, ... \]

In conclusion: the space harmonics of the flux density (magnetic field or air-gap magnetomotive force) corresponding to a two-phase armature, create rotating fields that depends inverse proportionally with the order of the harmonic. They are forward waves for \( \nu = 4k + 1 \) (that is 5, 9, 13…) and backward waves for \( \nu = 4k - 1 \) (that is 3, 7, 11…).

For \( \nu = 3 \), the field created by the two phases result as a sum:

\[ B_{\text{gec}}(\alpha, t) = B_{3\text{max}} \sin(\alpha - 3\nu \omega t) + \sin(\alpha + 3\nu \omega t) + \sin(\alpha - 3p\alpha + \pi) + \sin(\alpha + 3p\alpha - 2\pi) = B_{3\text{max}} \sin(\alpha + 3p\alpha) \]  

The result is a rotating field with a \( B_{3\text{max}} \) amplitude that acts backwards to fundamental:

\[ \Omega_{3} = -\frac{\omega}{3p} \]  

For \( \nu = 5 \), the resultant field is:

\[ B_{\text{gec}}(\alpha, t) = B_{5\text{max}} \sin(\alpha - 5\nu \omega t) \]  

The 5th order harmonics create a forward rotating magnetic field since:

\[ \Omega_{5} = \frac{\omega}{5p} \]  

In conclusion, the two-phase induction machines require the elimination of the 3rd order harmonics since they create the most important torque that acts in opposition to the main torque.

### 3 FEM analysis of the two-phase induction machine

A magnetostatic analysis (with specialized software – FLUX2D) has been performed. The induction machine, made in Germany, has the following rated parameters: \( m = 3 \) phases, \( 2p = 2 \), \( P_N = 0.55 \) kW, \( U_N = 380 \) V, Y connexion, \( I_N = 1.6 \) A, \( f = 50/60 \) Hz, \( n_N = 2850/3460 \) rot/min. Fig. 2a presents the cross-section of the motor. The principal geometrical parameters are: outer stator diameter – 130 mm; inner stator diameter – 69 mm; air-gap width – 0.3 mm; axial length of the magnetic circuit – 60 mm; stator slot number – 24; rotor slot number – 18.

Fig. 2a and 2b present the flux lines distribution and the air-gap flux density wave (including harmonic content) for the considered motor with cylindrical magnetic circuit and „normal” two-phase stator winding with \( q = Z_1/2p = 6 \) slots/pole/phase (symbol MCFC6). The results correspond to \( \omega t = 0 \) and the currents, given by (1), are equal to \( I_m = 2.66 \) A. The rotor is considered slotsless – a situation equivalent to synchronous speed operation (\( s = 0 \)). Fig. 3 gives the same results obtained for \( \omega t = 45^\circ \). It is to be pointed out that the 3rd order harmonics represents \( 7-10\% \) from fundamental. This fact has a negative influence on the motor operation and sometimes gives a backward rotation tendency to the rotor.
For the diminution of these harmonics, a different solution is proposed: a stator winding with \( q = 8 \) slots/pole/phase. This construction is possible if \( 2 \times 4 = 8 \) slots shelter turns of both phases (symbol MCFC8). These slots, which are symmetrically placed in the magnetic circuit, and form 4 zones of 2 slots must have a two times greater section in comparison with the others. The increase of the slot section must be realised only on radial direction, (the adjacent teeth section remains constant) and consequently the yoke section decreases significantly. The effects are considerable negative. To avoid this fact, a theoretical solution could consist in replacing the original turns with other turns having a smaller cross-section and a two-times greater density current. This was the

Fig 2: Cylindrical stator and slotless rotor (idealized situation, \( s = 0 \)), \( \omega t = 0^\circ \) (MCFC 6 X 2,66 A-0°)

Fig 3: Cylindrical stator and slotless rotor (idealized situation, \( s = 0 \)), \( \omega t = 45^\circ \) (MCFC 6 X 2,66 A-45°)
hypothesis that we assumed for the results presented in Fig. 4 (ωt=0°) and Fig. 5 (ωt=45°). Since the linear current density must remain constant, the current value is $I_m=2\text{A}$.

The following aspects worth to be mentioned: the fundamental amplitudes are the same for the two solutions and equal to 1.04 T. However, the 3rd order harmonics amplitude decrease for the MCFC8 to 2% from fundamental. This result, which is quite encouraging, led us to a practical solution. Fig. 6 (ωt=0°) and Fig. 7 (ωt=45°) present the results obtained for a machine with a geometry slightly modified. The cylindrical stator becomes a square (the former outer diameter...
represents the dimension of the square – symbol MPFC8). The higher slots are placed in the corners of the square. Thus, the slots shelter a two times greater number of turns, the linear current density remains constant and the decrease of the yoke is acceptable. The results put in view the following aspects: for $\omega t=0^\circ$, the fundamental increases with 5% (1.09 T) and the 3rd harmonic amplitude is 1.3% from fundamental.

Fig 6: Square stator and slotless rotor (proposed solution, $s=0$), $\omega t=0^\circ$ (MPFC 8 X 2A-0$^\circ$)

Fig 7: Square stator and slotless rotor (proposed solution, $s=0$), $\omega t=45^\circ$ (MPFC 8 X 2A-45$^\circ$)

These results prove the superiority of the new solution. The explanation is given by the presence of the square corners that avoid a supplementary saturation of the magnetic circuit. For $\omega t=45^\circ$ one notices an increase of the fundamental with 3% but also a rising of the 3rd order
harmonic to 8% from fundamental. A deeper investigation shows that this rising is given by the presence of a time harmonic of the current, which appears because of the saturation. An analytical explanation [6] starts from the magnetic flux – current [\(\varphi(i)\)] curve that has a flat region with saturation. As regards the flux density values, we find up to 2.17 T in the magnetic circuit.

In order to have „normal” values (up to 1.8 T), we have taken into discussion a smaller current (\(I_m=1A\)) and rotor with slots – symbol MPCC8.

![Fig 8: Cylindrical stator and rotor with slots (real solution), \(\omega t=30^\circ\) (MCCC 6 X 1.33A-30\(^\circ\))](image)

![Fig 9: Cylindrical stator and rotor with slots (improved solution), \(\omega t=15^\circ\) (MCCC 8 X 1A-15\(^\circ\))](image)

Fig.8 presents the obtained results for the machine with \(q=6\) and \(I_m=1.33A\) (MCCC6, corresponding to \(\omega t=30^\circ\)). The 3\(^{rd}\) order space harmonic represents more then 10% from fundamental. Fig. 9 presents the results for MCC8 with \(q=8\) and \(I_m=1A\) (corresponding to \(\omega t=15^\circ\)). In this case, the 3\(^{rd}\) order harmonic practically vanishes (value under 1%). Finally, Fig. 10 presents the results for MPCC8 – proposed solution – with \(q=8\) and \(I_m=1A\) (corresponding to
$\omega t=45^\circ$). Again, both space and time harmonics of the 3rd order vanish (no saturation). There is also a decrease of the maximum value of the flux density (1.67 T).

4 Conclusions

- The two-phase induction machines require a diminution of the 3rd order harmonic because of its negative effects.
- A possible solution (proper for two-pole machine) consists in the presence of slots with increased area, which are placed in equidistant positions.
- This solution is suitable for a stator made of square laminations.
- If the machine operates under saturation, then the solution looses its efficiency because of time harmonics of the currents.

References